2015

Bachelor of Science (B.Sc.) Semester—IV Examination

MATHEMATICS

(M_7–Partial Differential Equations and Calculus of Variation)

Paper—I

Time—Three Hours] [Maximum Marks—60

N.B. :— (1) Solve all the **FIVE** questions.

(2) All questions carry equal marks.

(3) Questions **1** to **4** have an alternative. Solve each question in full or its alternative in full.

UNIT—I

1. (A) Find the integral curves of the equations :

$$\frac{dx}{xz-y} = \frac{dy}{yz-x} = \frac{dz}{1-z^2}.$$ 6

(B) Solve the equation :

$$(x^2z - y^3)\,dx + 3xy^2dy + x^3dz = 0,$$

first showing that it is integrable. 6

OR

(C) Prove a necessary and sufficient condition that the Pfaffian differential equation $\vec{X} \circ d\vec{r} = 0$ should be integrable is that $\vec{X} \circ \operatorname{curl} \vec{X} = 0$. 6

(D) Elimate the arbitrary function f from the equation :

$$f(x^2 + y^2 + z^2,\ x + y + z) = 0.$$ 6

QUESTION ...

UNIT—II

(A) Find the general integral of the linear partial differential equation :

$$(y + zx)p - (x + yz)q = x^2 - y^2.$$ 6

(B) Find the integral surface of the linear partial differential equation :

$$x(y^2 + z)p - y(x^2 + z)q = (x^2 - y^2)z,$$

which contains the straight line $x + y = 0, z = 1.$ 6

OR

(C) Show that the equations $f(x, y, p, q) = 0$, $g(x, y, p, q) = 0$ are compatible if

$$\frac{\partial(f, g)}{\partial(x, p)} + \frac{\partial(f, g)}{\partial(y, q)} = 0$$

Verify also that the equations $p = P(x, y)$, $q = Q(x, y)$ are compatible if $\frac{\partial P}{\partial y} = \frac{\partial Q}{\partial x}$. 6

(D) Find the complete integral of $z^2 = pq\,xy$ by using Charpit's method. 6

UNIT–III

3. (A) Solve $xyr + x^2 s - yp = x^3 e^y$, where $r = \frac{\partial^2 z}{\partial x^2}$, $s = \frac{\partial^2 z}{\partial x \partial y}$ and $p = \frac{\partial z}{\partial x}$. 6

(B) Solve $(2D^2 - 5DD' + 2D'^2)z = 24(y - x)$, where $D \equiv \partial/\partial x$ and $D' \equiv \frac{\partial}{\partial y}$. 6

4

OR

(C) Solve : $\frac{\partial^2 z}{\partial x^2} - \frac{\partial^2 z}{\partial x \partial y} + \frac{\partial z}{\partial y} - z = \cos(x + 2y)$. 6

(D) Solve : $x^2 \frac{\partial^2 z}{\partial x^2} - y^2 \frac{\partial^2 z}{\partial y^2} - y \frac{\partial z}{\partial y} + x \frac{\partial z}{\partial x} = 0$

by using $u = \log x$ and $v = \log y$. 6

UNIT—IV

4. (A) Test for the extremum of the functional

$$I[y(x)] = \int_0^1 [x \sin y + \cos y]\, dx,\ y(0) = 0,\ y(1) = \pi/4$$

and show that the extremum can be found on the curve $y = \tan^{-1} x$. 6

(B) Find the shortest curve joining the two points (x_1, y_1) and (x_2, y_2) by using the functional

$$I[y(x)] = \int_{x_1}^{x_2} \sqrt{\left[1 + (y')^2\right]}\, dx$$

with $y(x_1) = y_1$ and $y(x_2) = y_2$. 6

OR

(C) Find the extremal of the functional :

$$I[y(x)] = \int_{x_0}^{x_1} \left[(y'')^2 - 2(y')^2 + y^2 - 2y \sin x\right] dx.$$ 6

(D) Find the extremals of the functional :

$$I = \int_1^2 \left[(z')^2 - x y' z\right] dx,\quad \text{given } y(1) = z(1) = 1,$$

$y(2) = -1/6$, $z(2) = 1/2$. 6

QUESTION—V

5. (A) Solve $xdx + ydy + zdz = 0$. 1½

(B) Eliminate the constants a and b from the equation $ax^2 + by^2 + z^2 = 1$. 1½

(C) Solve $yp + xq = zy$, where $p = \frac{\partial z}{\partial x}$ and $q = \frac{\partial z}{\partial y}$. 1½

(D) For the equations $xp = yq$, $z(xp + yq) = 2xy$, find $\frac{\partial(f, g)}{\partial(y, q)}$. 1½

(E) Solve $(D^2 - D'^2)z = 0$, where $D \equiv \frac{\partial}{\partial x}$ and $D' \equiv \frac{\partial}{\partial y}$. 4 1½

(F) Find PI of $\log s = x + y$, where $s = \frac{\partial^2 z}{\partial x \partial y}$. 1½

(G) If $I[y(x)] = \int_0^1 [y(x)]^2 \, dx$, then find $I[y_1(x)]$ if $y_1(x) = \sqrt{(1 + x^2)}$. 1½

(H) If F depends solely on y' i.e. $F = F(y')$, then by using Euler's equation, show that the extremals are straight lines. 1½